Appendix E
Bending by a Gaussian Electron Density Distribution

Suppose that the electron number density distribution $n_e(r)$ is locally spherical symmetric and Gaussian-distributed around some altitude $r_o - R_E$. Then $n_e(r)$ is given by

$$n_e(r) = \frac{\text{TEC}}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(r-r_o)^2}{2\sigma^2} \right) \tag{E-1}$$

where TEC is the total columnar electron content and σ provides the width of the distribution. Equation (E-1) can be considered as the leading term in a Chapman distribution of the form

$$n_e(r) = A \exp\left(1 - \frac{r-r_o}{\sigma} - \exp\left(\frac{r-r_o}{\sigma} \right) \right) \tag{E-2}$$

Using Eqs. (2.2-2) and (2.10-1), it follows that the bending-angle profile from this distribution is given by

$$\alpha(r) \doteq \frac{\kappa\, \text{TEC}}{\text{f}^2} \sqrt{\frac{2r}{\sigma^3}} I(\gamma) \tag{E-3}$$

where $I(\gamma)$ is given by

$$I(\gamma) = -\int_{\gamma}^{\infty} \frac{y}{\sqrt{y-\gamma}} \exp\left(-\frac{y^2}{2} \right) dy, \quad \gamma = \frac{r-r_o}{\sigma} \tag{E-4}$$

The integral $-I(\gamma)$ is shown in Fig. E-1. For $\gamma > 0$, the ray traverses only through the upper wing of the electron distribution, and α damps to zero rapidly with increasing altitude. For $\gamma < 0$, the ray traverses through the peak of the distribution twice. The zero crossing occurs at $\gamma \cong -3/4$; this is the altitude of the tangency point where the positive bending contribution from the lower wing of the electron density distribution just cancels the entire contribution from the upper wing. Below this altitude, the bending angle remains positive and damps to zero more gradually with decreasing altitude.

This electron distribution and the bending-angle profile are used in Chapter 5 in connection with a discussion there on multipath and caustics in a wave theoretic framework.

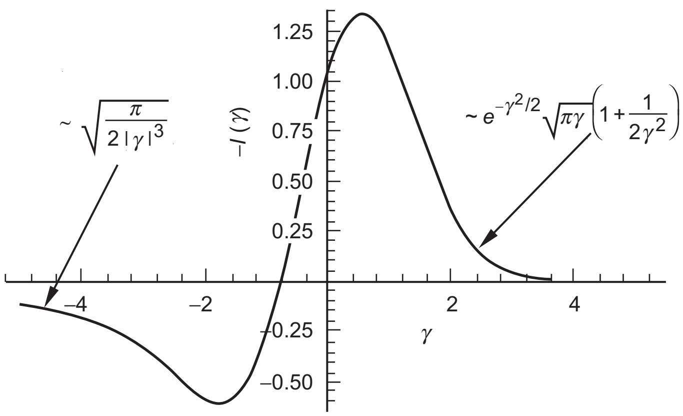

Fig. E-1. Normalized bending-angle profile for a Gaussian refractivity distribution.